Envy Free Cake Cutting

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Assumptions

- No arbitration – agreed procedure
- Ignorant of others' valuations
- Maximin objective
- Additive valuations
- Divisible goods

- Discrete or continuous cuts
- Arbitary or constrained cuts

Criteria

- Fair / proportional
- Envy free
- Exact
- Pareto optimal
- Equitable

Hugo Steinhaus

Stefan Banach

Bronislaw Knaster

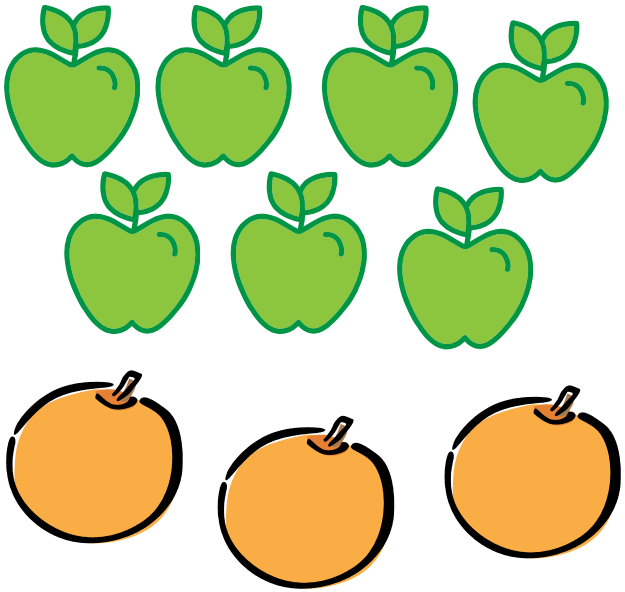
The Scottish Café, Lwów

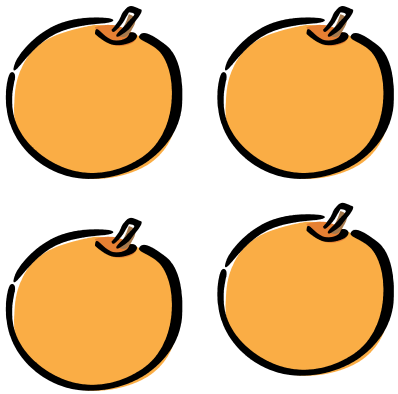
Cake from Poland

Divide and Choose

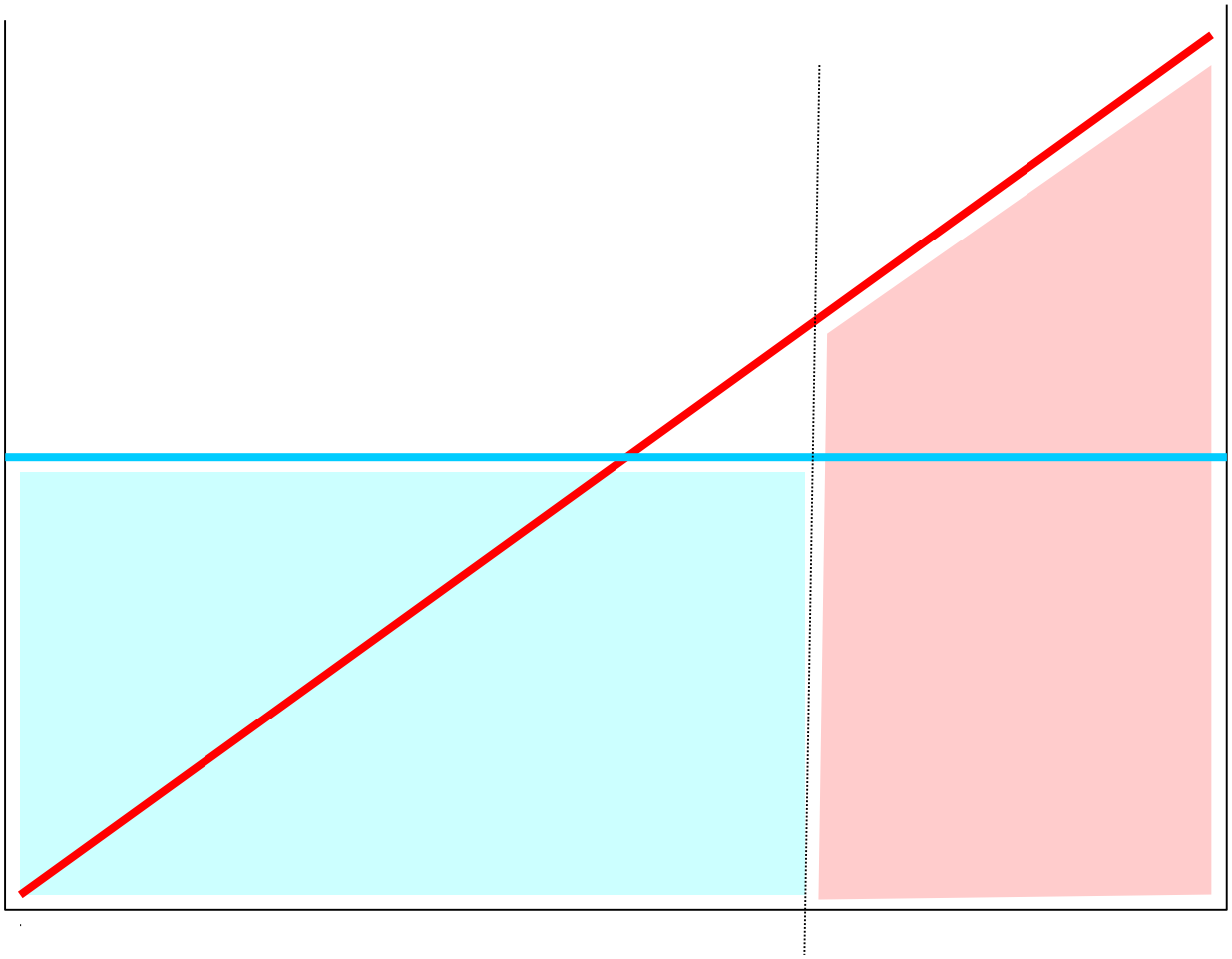
7 apples and 7 oranges

Alan likes apples, Bob likes oranges

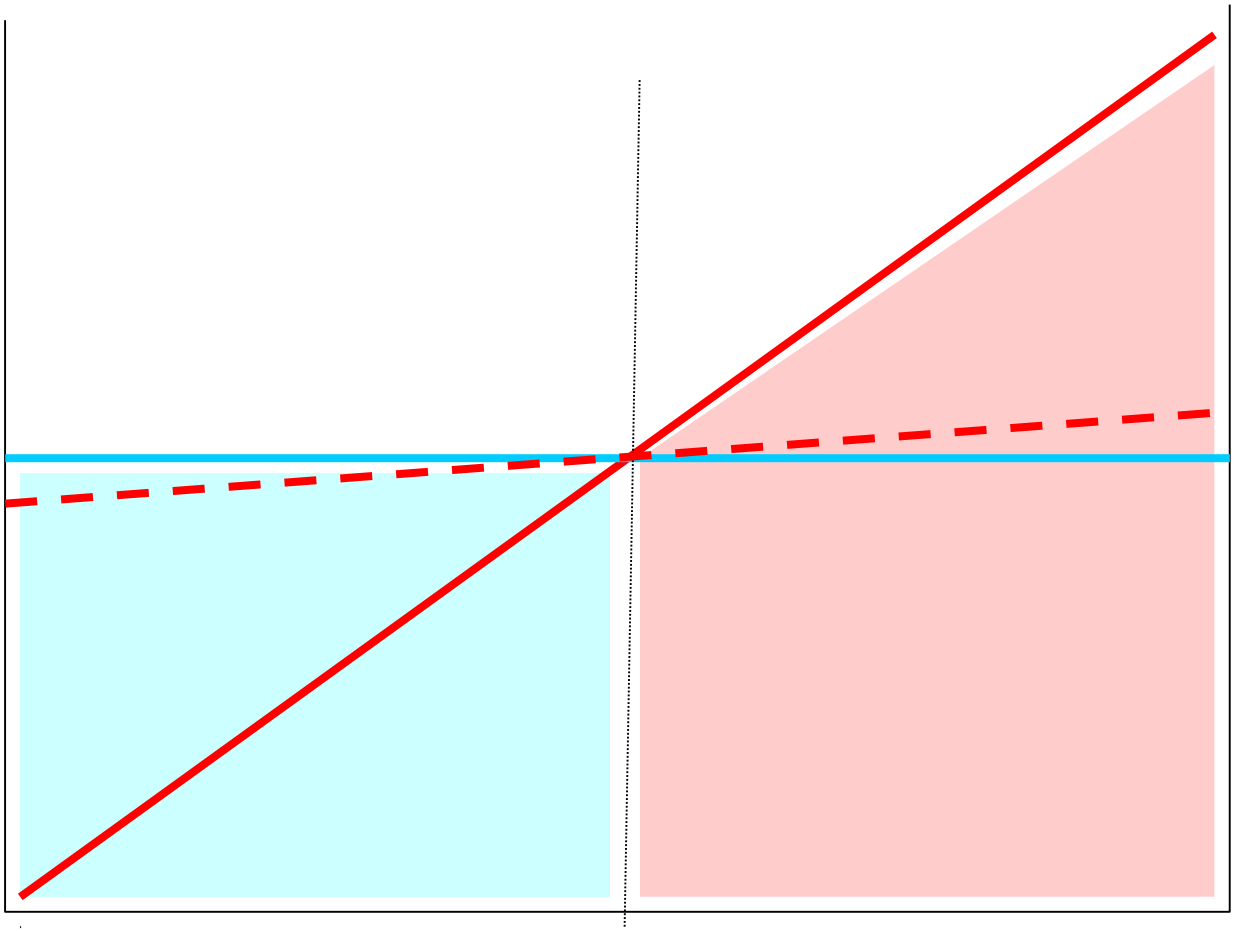




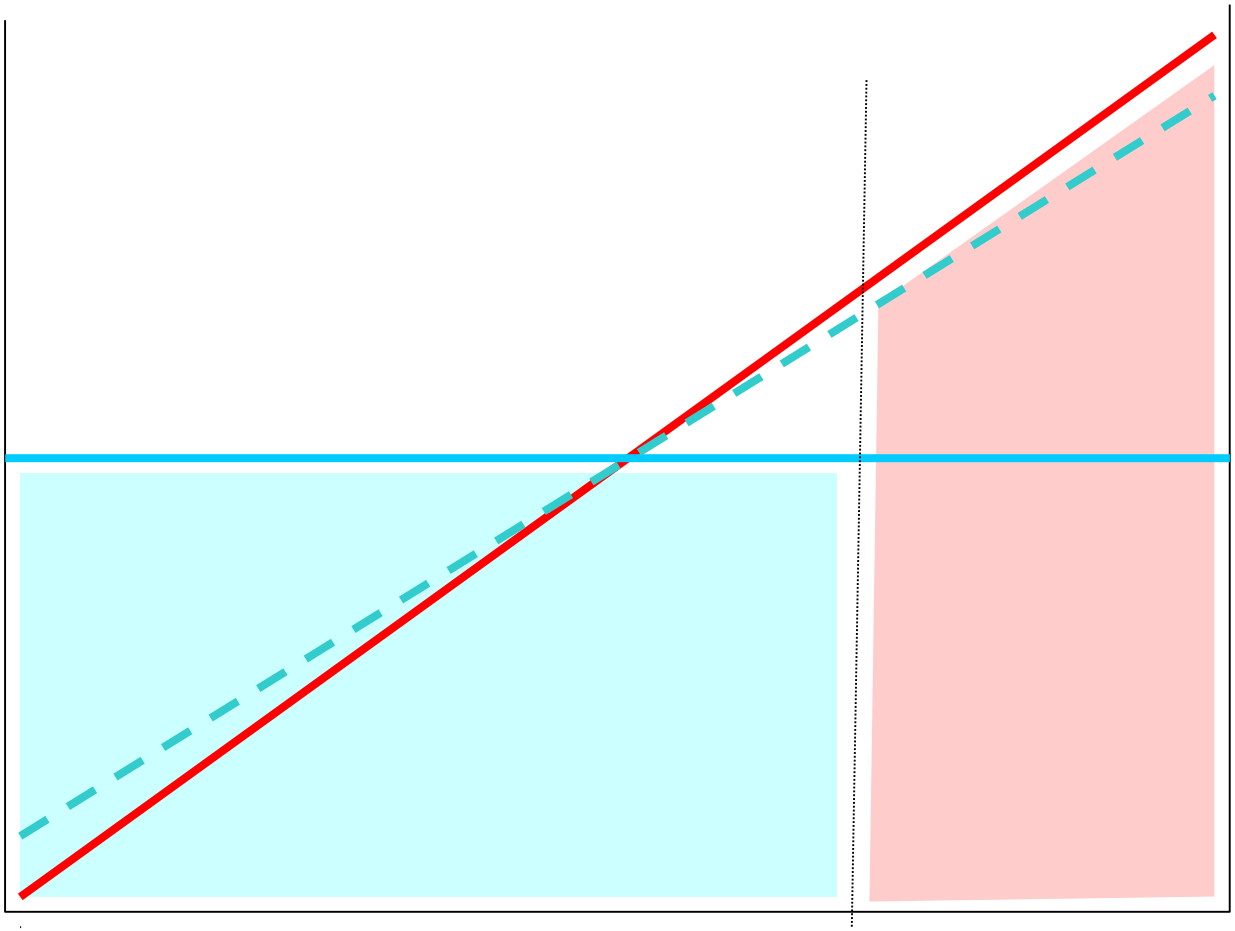
0.618 : 0.618



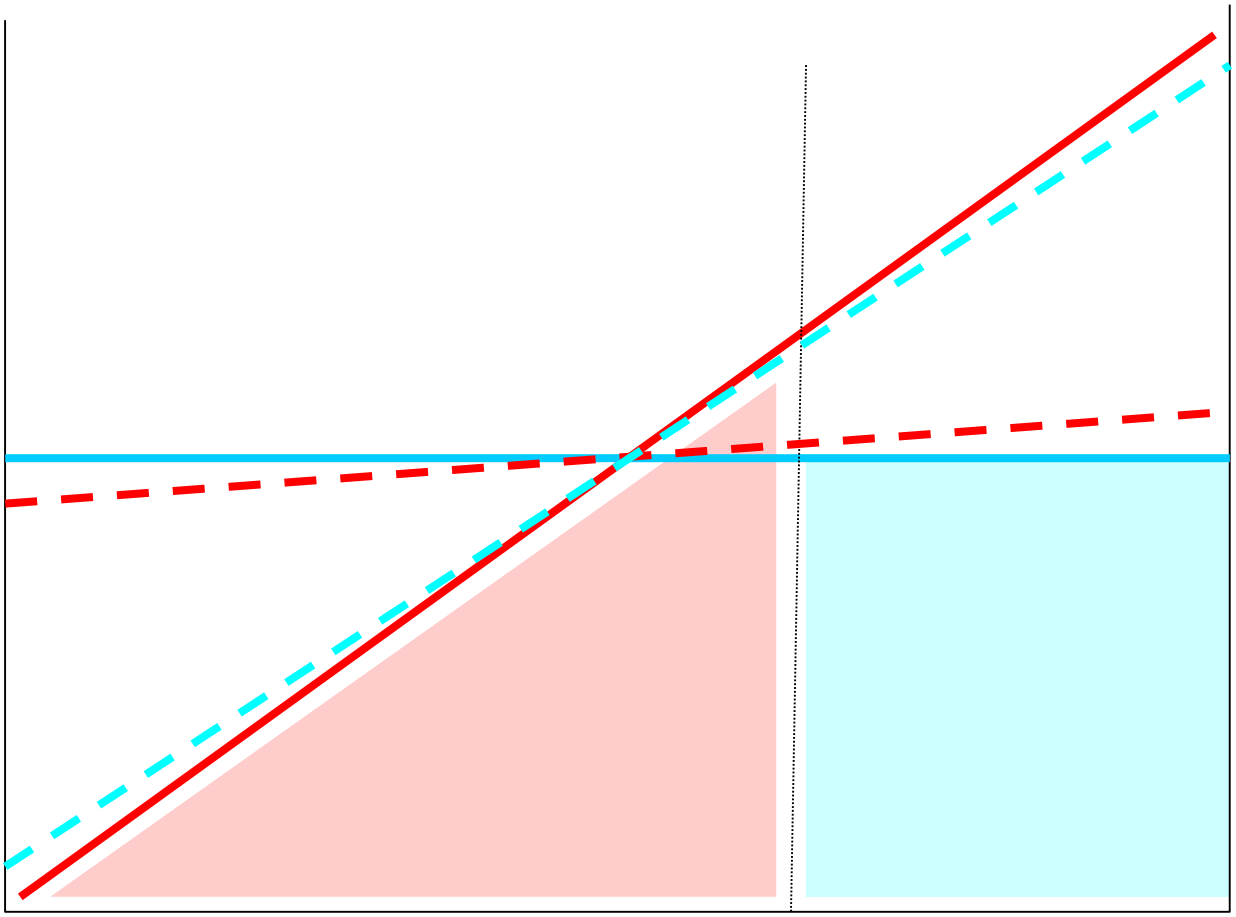
0.5 : 0.75



0.667 : 0.5



0.382 : 0.382



Exact Division

One person turns knife round so they think half the cake is on either side

After the knife is turned 180° the knife must be where it started

Second person stops the knife when the think half the cake on ether side

Bits allocated at random.

Intermediate value theorem

3-person envy-free division

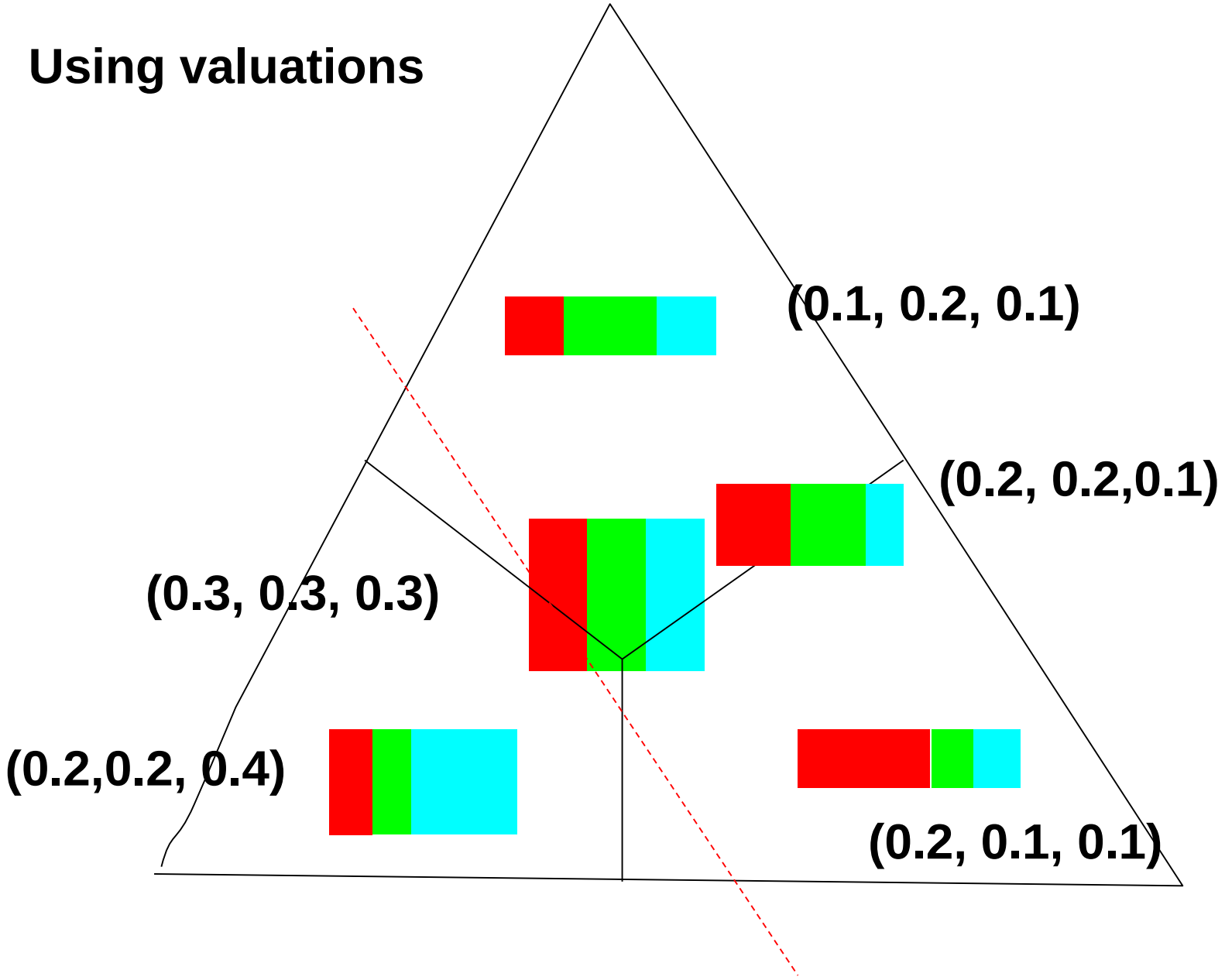
- Alice cuts cake in three
- Betty trims off a bit, so 2 biggest equal
- Carol selects
- Betty selects, trimmed piece if available
- Alice takes last piece
- Non trimmed of B,C: cuts trimmings
- Trimmed, Alice, Non-trimmed take piece

Due to Conway, Guy and Selfridge?

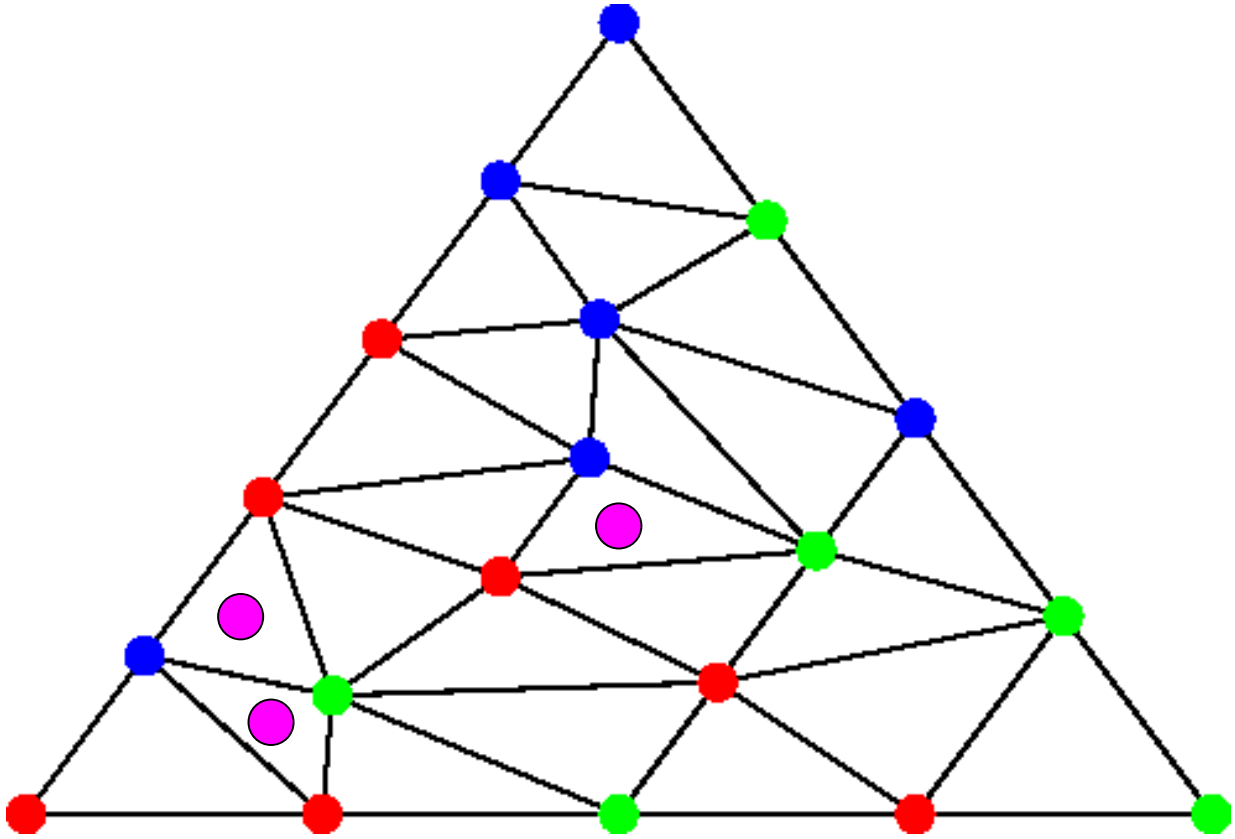
Others

- Divide and conquer fair division
- Stromquist moving knife - continuous
- Brams Taylor Zwicker moving knife
- Brams Taylor envy free any number
- Adjusted Winner – discrete+money
- Chore division – rents, dirty work
- Surplus procedure

Using valuations



Sperner's Lemma

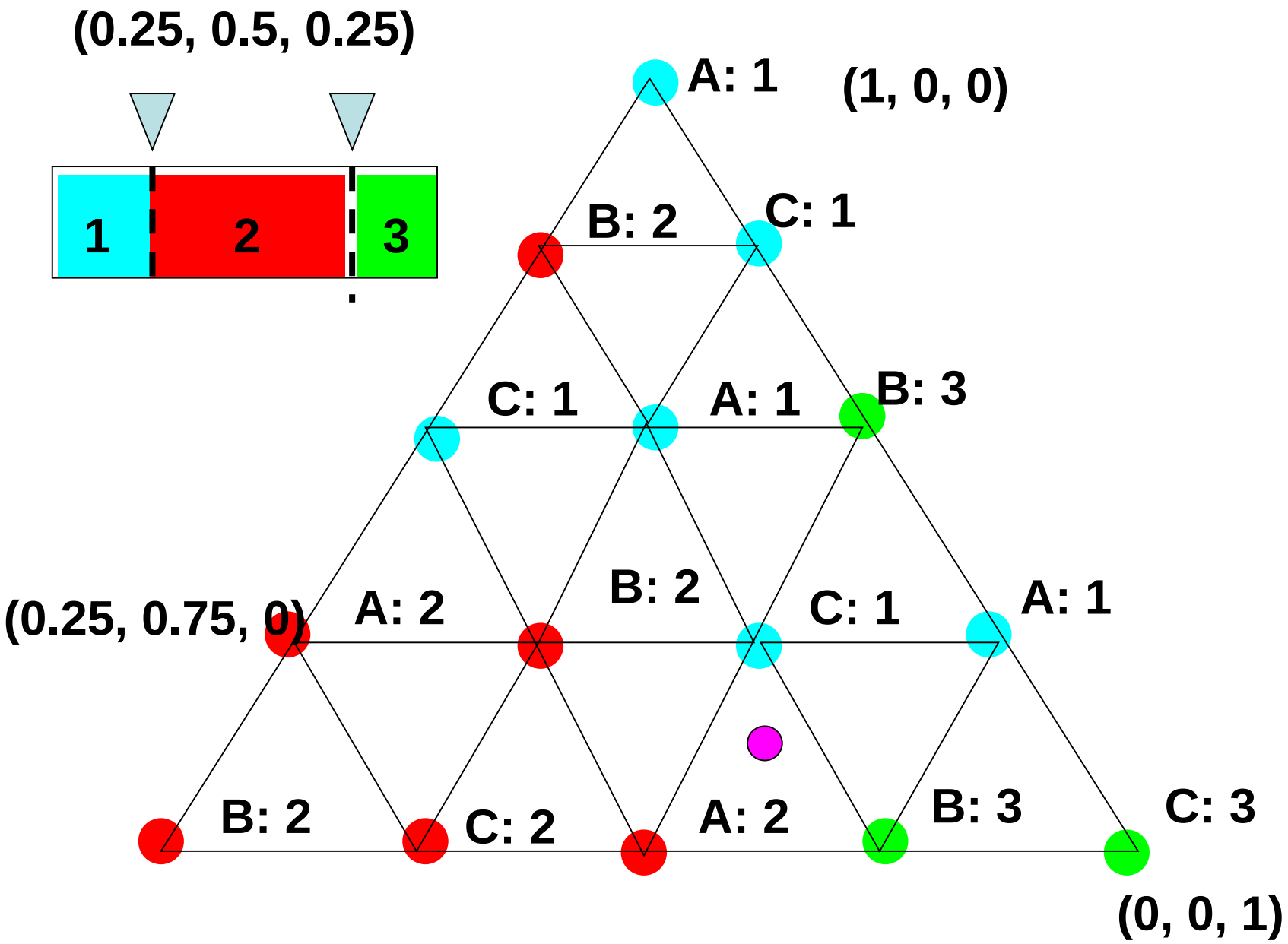


Emmanuel Sperner 1924 can be used to prove Brouwer's fixed point theorem

Envy Free using Sperners Lemma

- Envy-Free fair division devised by Forest Simmons
- Variant of Sperner's lemma by Herbert Scarf used by Francis Su for the rent division problem
- Algorithm for both on the web at

  www.math.hmc.edu/~su/fairdivision/calc/


(0.25, 0.5, 0.25)
1
2
3
A: 1
(1, 0, 0)
B: 2
C: 1
C: 1
A: 1
B: 3
(0.25, 0.75, 0)
A: 2
B: 2
C: 1
A: 1
B: 2
C: 2
A: 2
B: 3
C: 3
(0, 0, 1)

Envy free versus Equitability